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SHORT-RANGE CORRELATIONS IN CARBON-12, OXYGEN-16, AND NEON-20: INTRINSIC PROPERTIES

by Richard C. Braley, William F. Ford, Richard L. Becker, and Malcolm R. Patterson

Lewis Research Center
Cleveland, Ohio 44135

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16. Abstract

For the first time the Brueckner-Hartree-Fock (BHF) method has been applied to nuclei whose intrinsic structure is nonspherical. Reaction matrix elements were calculated as functions of starting energy for the Hamada-Johnston interaction using the Pauli operator appropriate to ^{16}O and a shifted oscillator spectrum for virtual excited states. Binding energies, single-particle energies, radii, and shape deformations of the intrinsic state, in ordinary as well as renormalized BHF, are discussed and compared with previous HF studies and with experiment when possible. Results are presented for ^{12}C, ^{16}O, and ^{20}Ne. It is found that the binding energies and radii are too small, but that separation energies are well reproduced when the renormalized theory is used.

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SUMMARY

The short-range correlations known to be present in nuclear matter are included in the calculation of properties of finite, nonspherical nuclear systems by employing the Brueckner-Hartree-Fock (BHF) method. The necessary modifications in this technique, which has previously been applied only to spherical nuclei, are presented. Renormalization resulting from inclusion of occupation probabilities is also discussed. Intrinsic spectra, radii, quadrupole moments, and hexadecapole moments are investigated for ^{12}C and ^{20}Ne, and the ^{16}O spherical results are presented. Predictions are included for the renormalized BHF as well as BHF.

INTRODUCTION

The connection between the nuclear many-body problem and the prediction of properties of finite nuclei has been studied with increased intensity in the last few years (refs. 1 to 3). Advances in computational technology have made it possible to drop the "closed-shell-core" assumption which was common to most nuclear structure calculations (refs. 4 to 6). Thus it has become feasible to attempt to understand nuclear phenomena in terms of a microscopic theory without the additional uncertainties which result when an inert core is assumed. (The inert-core effects are usually manifested through the need for effective charges in calculating electromagnetic properties and the need for effective interactions in obtaining matrix elements of the nuclear Hamiltonian.)

A natural starting point for many microscopic studies of nuclear systems has been the Hartree-Fock (HF) method (ref. 7), which has contributed significantly to our understanding of the properties of nuclei (ref. 8). The use of the HF method has been limited

*Oak Ridge National Laboratory, Oak Ridge, Tennessee.

somewhat by the type of interactions available for use in calculations, but it is certainly the most convenient method presently available for investigating nuclear many-body systems. For the most part HF studies have been restricted to the use of (1) phenomenological effective interactions (refs. 9 and 10) or (2) effective interactions based directly or indirectly on nuclear matter calculations (refs. 11 to 13). Microscopic studies of both spherical and deformed nuclei have been made using such effective interactions.

Bassichis and collaborators (ref. 10) have investigated many aspects of spherical nuclei and deformed intrinsic systems using the HF method with the bare Tabakin interaction. This interaction fits the low energy nucleon-nucleon scattering data but yields poor saturation properties when used in structure studies. Large second-order corrections to the energy indicated that a Brueckner-Goldstone series would be desirable, and Gmitro and Sotona (ref. 14) made such a calculation for ^{16}O and ^{4}He and found considerable improvement in the binding energy. Although they did not calculate the rms radius, it is expected to be smaller than experiment. On the other hand, Ford, Braley, and Bar-Touv have found that many other nuclear properties (such as energy spectra, E2 rates, quadrupole moments, and inelastic proton scattering cross sections) can be well described when the bare Tabakin force is used in microscopic studies (ref. 8).

Effective interactions of the second type have been constructed using realistic nucleon-nucleon forces, such as the Yale and Hamada-Johnston potentials. The effective interaction of Shakin et al. (ref. 11) has been used in several microscopic studies of spherical and deformed nuclei. Pal and Stamp have investigated intrinsic HF properties with it and found that saturation could not be obtained (ref. 15). However, Gunye successfully used the same matrix elements to study energy levels and electromagnetic properties of several nuclei in the s-d shell (ref. 16). Davies, Krieger, and Baranger have made HF calculations (ref. 17) using a velocity-dependent two-body force which saturates nuclear matter (ref. 13). They found fairly good agreement with experiment for binding energies, energy levels, and density distributions for several nuclei which they considered to be spherical. Later deformed HF calculations by Krieger were made using a similar force, but the results were not as impressive (ref. 18). Lee and Cusson (Chalk River Nuclear Laboratory) have used the semirealistic interaction of Saunier and Pearson in recent unrestricted HF calculations (private communication). They studied a large number (57) of light nuclei and generally found quite good agreement with experiment.

The effective interaction of Negele (LINEG) was also constructed from nuclear matter considerations (ref. 12). This interaction, which is density- and energy-dependent, was found to yield very good binding energies, sizes, single-particle energies, and electron scattering cross sections for several spherical nuclei. Much of the improvement in binding energy and radius, however, is due to adjustment of the strength and range of the interaction to obtain the desired saturation properties of nuclear matter or of selected finite nuclei.

Zofka and Ripka have used LINEG to study the intrinsic states for the light doubly even N=Z nuclei (ref. 19). Their results are quite good, but it should be kept in mind that they are calculated for, or deduced from, the intrinsic state and therefore may be altered by angular momentum projection.

Predictions similar to those of Zofka and Ripka have been reported by Reid et al. (ref. 20) who claim to have done a BHF calculation. Theirs, however, is not a Brueckner-Hartree-Fock calculation, but rather is similar to that of Zofka and Ripka in which a density dependent effective interaction is used.

Although the HF studies using effective interactions have significantly increased our understanding of the structure of nuclei, one would nevertheless like to begin with a realistic nucleon-nucleon interaction and, with as few approximations as possible, calculate nuclear properties based on a many-body theory. The Brueckner-Goldstone (BG) theory provides a starting point for such calculations.

Following the pioneering work of Brueckner (ref. 21) and Goldstone (ref. 22), a large number of papers were written to clarify the theory and extend it beyond applications to nuclear matter (refs. 23 to 27). In recent years, the usefulness of the nuclear-matter calculations in studying properties of finite nuclei was demonstrated by Kuo and Brown (ref. 5), and the development of the Brueckner theory for closed-shell nuclei was advanced considerably by Kohler and McCarthy (ref. 28) and by Becker, MacKellar, and Morris (ref. 29).

Uncertainties do remain in the many-body theory because of a lack of understanding of the two-body force and some questions regarding the importance of higher-order Brueckner-Goldstone diagrams (ref. 30) and three-body clusters (ref. 31). However, if one hopes to understand the structure of nuclei in terms of a true microscopic picture, a Brueckner-Hartree-Fock calculation appears to be the most reasonable approach presently available.

The application of the BHF method to the investigation of finite-nucleus properties has thus far been limited to spherical, closed-shell nuclei. Among the most significant of the calculations made to date have been those of the Oak Ridge groups (refs. 32 to 38). The self-consistent BHF studies by Davies et al. (refs. 35 and 36) demonstrated the ease and reliability with which such calculations could be made, although they found the nuclei to be too small and underbound. The inclusion of occupation probabilities in finite-nucleus Brueckner calculations by Becker made a substantial improvement in the single-particle energies, and it was found that the depletion factors for normally occupied single-particle orbits were of the order of 15 percent. Subsequent calculations by Davies and McCarthy showed that good results for the binding energy could be obtained when such higher-order diagrams are included; however, the nuclei were still too small.

The results discussed previously indicate that the nuclear many-body problem is now sufficiently understood to make it reasonable to examine the degree to which one can use it to predict nuclear properties while, at the same time, attempting to refine and extend

our present knowledge of the problem. Since the majority of nuclei are not of the spherical, closed-shell type, it is of interest to determine whether the energy-dependent reaction matrix obtained from BHF calculations leads to deformations, gaps, etc. which are similar to those resulting from standard HF calculations with effective interactions. The light deformed nuclei provide a good starting point for such a study, since the number of particles involved is small enough to keep the problem tractable and the simple HF approximation is understood well enough to provide guidelines for what is recognized to be a rather complicated problem. Although there are some conceptual difficulties associated with obtaining physical states from deformed intrinsic systems when BHF is used, there is still much to be gained from a study of properties of the intrinsic system. For instance, it would be interesting to see how deformations, single-particle energies, and energy gaps are affected when one does BHF and renormalized BHF rather than the simpler HF calculation using an energy independent interaction. The aim of this report is to discuss and compare such calculations for ^{12}C, ^{16}O, and ^{20}Ne.

THEORY

The deformed intrinsic HF state of a many-body system is nondegenerate in the body-fixed frame; therefore, in this frame Goldstone's linked-cluster perturbation expansion (ref. 30) is valid. If one makes the usual association of terms in the perturbation series with Brueckner-Goldstone diagrams, then - from the viewpoint of diagram analysis - the only departure from the more familiar spherical situation is that propagation lines are now associated with deformed single-particle states. It follows that summations and cancelations of particular diagrams are identical for finite spherical and deformed systems. As usual, a summation of terms representing a series of ladder diagrams corresponds to (and is accomplished by) solving the integral equation

$$t_{12}(E_s) = v_{12} - v_{12} \frac{Q}{h_{12} - E_s} t_{12}(E_s) \tag{1}$$

Here $t_{12}(E_s)$ is Brueckner's reaction matrix, v_{12} is a nucleon-nucleon interaction with short-range repulsion, E_s is the "starting energy" (ref. 29), Q is the Pauli operator which forbids scattering into occupied orbitals in the nucleus being considered, and $H_{12} = h_1 + h_2$. The single-particle Hamiltonians h_1 and h_2 include potentials which are defined so as to cancel certain classes of diagrams; these potentials are deformed for the case of interest here.

Because of the hard core in v_{12}, it is convenient to define a correlated two-particle wavefunction ψ_{12} by means of the relation

$$t_{12}(E_s)\varphi_{12} = v_{12}\psi_{12} \tag{2}$$

from which one obtains the Bethe-Goldstone equation

$$\psi_{12}(E_s) = \varphi_{12} - \frac{Q}{h_{12} - E_s} v_{12}\psi_{12}(E_s) \tag{3}$$

The direct solution of this equation is very difficult because the Pauli operator Q and the Green's function $\left(h_{12} - E_s\right)^{-1}$ depend on nucleon orbitals which are not determined until the HF problem is solved using the matrix elements of $t_{12}(E_s)$. This aspect of self-consistency is usually treated indirectly by defining a "reference" t-matrix $t^R_{12}(E_s)$ which satisfies

$$t^R_{12}(E_s) = v_{12} - v_{12} \frac{Q^R}{h^R_{12} - E_s} t^R_{12}(E_s) \tag{4}$$

as a first approximation to the nuclear t-matrix. The reference t-matrix is found from the corresponding reference Bethe-Goldstone equation

$$\psi^R_{12}(E_s) = \varphi_{12} - \frac{Q^R}{h^R_{12} - E_s} v_{12}\psi^R_{12}(E_s) \tag{5}$$

and the nuclear t-matrix then obtained - after the HF problem is solved using the reference t-matrix - by means of the relation

$$t_{12}(E_s) = t^R_{12}(E_s) - t^R_{12}(E_s)\left(\frac{Q^R}{h^R_{12} - E_s} - \frac{Q}{h_{12} - E_s}\right) t_{12}(E_s) \tag{6}$$

Now it is clear that solution of equation (5) will also be complicated if h_{12} contains deformed single-particle potentials. This is immediately obvious if we contemplate the transformation to relative and center-of-mass coordinates, which is necessary since v_{12} is given in that representation. On the other hand, if equation (5) is solved for spherical reference operators h^R_{12} and Q^R this difficulty will be avoided, and the problem of making Pauli corrections (to Q^R) and spectral corrections (to h^R_{12}) by using

equation (6) will also be greatly simplified. (This is essentially the procedure we have in mind.)

Once the reaction matrix elements have been obtained, the Hartree-Fock part of the BHF calculation begins. The set of self-consistent equations to be solved are

$$h|\lambda\rangle = e_\lambda|\lambda\rangle \tag{7}$$

where

$$\langle\lambda|h|\mu\rangle = \langle\lambda|K|\mu\rangle + \langle\lambda|U|\mu\rangle \tag{8}$$

Here K is the kinetic energy operator, and U is a one-body potential whose matrix elements are obtained from those of $t_{12}(E_s)$ by means of an equation in which the value of the starting energy E_s depends on whether λ and μ refer to hole-hole, hole-particle, or particle-particle states (refs. 35 and 37). This results in a double self-consistency requirement: the orbitals $|\lambda\rangle$ are eigenstates of $(K + U)$, and U depends on the energies of the filled orbitals. It therefore appears that, in order to do BHF correctly, we must recalculate the reaction matrix after each iteration in a self-consistency procedure. Fortunately, this procedure can be avoided by the technique introduced by McCarthy and Davies (ref. 37) in which t is expressed as a power series:

$$t_{12}(E_s) = \sum_{n=1}^{N} A_n E_s^n \tag{9}$$

If the reaction matrix is calculated for several starting energies, the coefficients A_n may be obtained by a suitable fitting procedure. Once the reaction matrix is obtained as a function of starting energy, it would be possible after each iteration in the BHF problem to use equation (6) to make the Pauli corrections. In practice, one would probably get sufficiently accurate results if these corrections were only made for the last two or three steps in the iterative procedure.

The usual method of solving equations (7) is to introduce a finite set of basis vectors which span several major shells. The equations then become

$$\sum_b \left\{\langle a|K|b\rangle + \langle a|U|b\rangle\right\} C_b^\lambda = e_\lambda C_a^\lambda \tag{10}$$

where the coefficients C_k^λ express the deformed orbitals in terms of a spherical oscillator representation, that is,

$$|\lambda\rangle = \sum_{k} C_k^{\lambda} |k\rangle \tag{11}$$

The method of Davies, Baranger, Tarbutton, and Kuo (ref. 35) may be used to obtain the matrix elements of U from those of $t_{12}(E_s)$:

$$\langle a|U|b\rangle = \frac{1}{2} \sum_{a'cd} \sum_{\xi} \sum_{\lambda}^{occ.} \rho_{aa'}^{\xi} \rho_{dc}^{\lambda} \left\langle a'c|t_{12}^{A}(\epsilon_{\lambda\xi})|bd\right\rangle$$

$$+ \frac{1}{2} \sum_{b'cd} \sum_{\eta} \sum_{\lambda}^{occ.} \rho_{bb'}^{\eta} \rho_{dc}^{\lambda} \left\langle ac|t_{12}^{A}(\epsilon_{\lambda\eta})|b,d\right\rangle \tag{12}$$

where

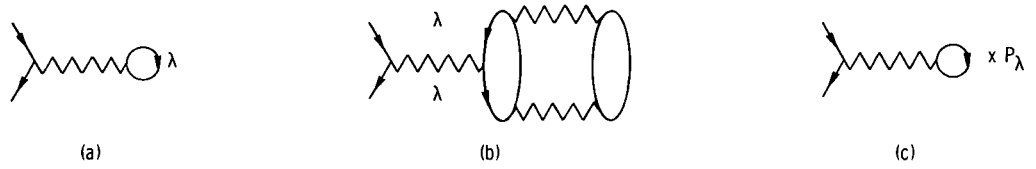
$$\rho_{ab}^{\lambda} = C_a^{\lambda *} C_b^{\lambda}$$

Although this expression was derived using an approximation for the starting-energy dependence of particle-particle and particle-hole matrix elements, it was pointed out by Davies et al. (ref. 37) that the effect on states of primary interest (occupied) is of second order only. It was later demonstrated by Davies and Baranger (ref. 36) that the exact expression gives essentially the same results for light nuclei. In the case of a spherical, closed-shell nucleus, the evaluation of equation (12) is straightforward, because the summations over magnetic quantum numbers may be carried out in closed form. With deformed nuclei this is not possible; thus, the effective dimensionality of the basis space is greatly increased. Since the summations in equation (12) must be repeated for each iteration (because the $\epsilon_{\lambda\mu}$ and ρ_{ab}^{λ} have changed), great care must be taken to keep the time required per iteration within reasonable limits. Once the iterative procedure has converged to a self-consistent set of orbitals $|\lambda\rangle$, the (first-order) binding energy is given (as in HF theory) by

$$E_1 = \frac{1}{2} \sum_{\lambda}^{occ.} \left[\langle\lambda|K|\lambda\rangle + e_{\lambda}\right] \tag{13}$$

In contrast to ordinary BHF, the renormalized theory (RBHF) takes into account the depletion of normally occupied single-particle states resulting from two-nucleon correlations (ref. 3). Probably the main difference is that RBHF includes, along with the

usual BHF contribution to the one-body potential (diagram (a)), the contribution of diagram (b)



(a) (b) (c)

which, in the unrenormalized Brueckner theory, is regarded as one of two third-order rearrangement potentials. Brandow (ref. 26) urged that all such potentials be included in the definition of the self-consistent field, where they would renormalize the BHF term, rather than be calculated only as a rearrangement correction (ref. 39). Thus, in the RBHF approximation, the one-body potential is represented by diagram (c), which includes the contribution of diagram (a) and (b) and a host of similar but higher-order diagrams. That series of diagrams can be summed as before by appealing to the integral equation which the series represents. The result may be written as diagram (a) times a factor P_λ (diagram (c)), where P_λ is the occupation probability for the orbit $|\lambda\rangle$. Practically speaking, the effect on our calculation is to require insertion of the appropriate factors of P_λ in equation (12) and the use of a modified expression for the binding energy:

$$E_1 = \frac{1}{2} \sum_{\lambda} \left[\langle \lambda | K | \lambda \rangle \, P_\lambda + e_\lambda (2 - P_\lambda) \right] \tag{14}$$

A complete renormalization of the entire perturbation series has been obtained by Brandow and is discussed in detail in reference 3.

RESULTS

Reaction matrix elements were calculated as functions of starting energy for the Hamada-Johnston interaction (ref. 40), using a Pauli operator (Q) appropriate to ^{16}O and a shifted oscillator spectrum (h_{12}) for the virtual excited states. Detailed descriptions of the method used to solve the reference Bethe-Goldstone equations, as well as a discussion of the need for a shifted spectrum, may be found in reference 29. The t-matrix so obtained was the spherical reference t-matrix with which the deformed Hartree-Fock calculations were begun, that is, the oscillator representation of the t-matrix elements as they appear on the right side of equation (12). At each subsequent iteration in the solution

of the HF equations, the starting energies from the previous iteration were used in equation (12) to obtain the one-body potential; however, the Pauli corrections were not made. The "on-shell" prescription is used for the particle-particle matrix elements in the deformed BHF calculations, although an approximate "off shell" prescription was used in the original spherical calculations (refs. 29 and 35). The choice of the shift parameter C used in the calculations has been discussed in reference 27.

Properties of the intrinsic states of ^{12}C, ^{16}O, and ^{20}Ne have been calculated for oscillator lengths of 1.57 and 1.77 femtometers using BHF and renormalized BHF. The variational problem was solved subject to the condition that the deformed intrinsic states possess axial symmetry and four-fold degeneracy. Expectation values of operators are calculated with the uncorrelated wavefunctions. The results have not been corrected for Coulomb and center-of-mass effects; such corrections should be small (1 to 2 MeV) for the nuclei studied here.

The HF calculations with which our results are compared are those of Zofka and Ripka (ref. 19), who used the effective interaction of Negele. This interaction is a density-dependent one which was found to yield good binding energies, sizes, single-particle energies, and electron scattering cross sections for certain spherical nuclei. Much of the improvement in binding energy and radius, however, is due to adjustment of the strength and range of the interaction to obtain the desired saturation properties of nuclear matter or selected finite nuclei. In the tables contained in this report, ZR refers to the results labelled II in reference 19. For this set of calculations, perturbation theory was used to correct for the fixed starting energy and Coulomb interaction. Whenever possible, the ZR values appropriate to neutrons are quoted, since we do not treat the Coulomb interaction in the calculations reported here.

The results for ^{12}C are presented in table I. As expected from our experience with spherical, closed-shell nuclei, the RBHF calculation yields better values than the BHF calculation for both oscillator lengths. An oblate shape for the nucleus is the lowest energy solution predicted, in agreement with the usual HF studies. Note that the radius predicted by the ZR calculation is considerably larger than experiment, and that their intrinsic quadrupole moment is 30 percent larger than the largest RBHF result. This is not surprising since the radius given by RBHF is only 2.37 femtometers, as compared to 2.59 femtometers given by ZR. We also observe that deformation increases when occupation probabilities are included and that the level ordering is unchanged. In all cases, the nucleus is underbound, although ZR's results are 20 percent larger than the RBHF results. The intrinsic properties of ^{12}C do not seem to be very sensitive to the oscillator length. Proton separation energies are given in the experiment column. Results are given for neutrons in the (BHF, RBHF) calculations, but the Coulomb effects should only increase the results by about 2 MeV (i.e., decrease the binding).

TABLE I. - COMPARISON OF BHF AND RBHF PREDICTIONS OF INTRINSIC PROPERTIES OF ^{12}C WITH THE RESULTS OF ŽOFKA AND RIPKA

Properties	Oscillator length, b, fm					
	1.57		1.77		1.67	
	Calculation					
	BHF	RBHF	BHF	RBHF	ZR[a]	Experiment[b]
Binding energy per nucleon, -E/A, MeV	4.6	5.6	4.5	5.2	6.4	7.7
Energy gap, Δ, MeV	12.7	11.2	11.9	10.4	9.8	
Intrinsic rms radius, $\langle R^2 \rangle^{1/2}$, fm	2.17	2.30	2.25	2.37	2.59	2.40±0.03
Mass quadrupole moment, $\langle Q_2 \rangle$, fm^2	-25.4	-29.6	-26.6	-30.7	-40.9	
Mass hexadecapole moment, $\langle Q_4 \rangle$, fm^4	17.7	23.4	19.1	26.7		
(Magnetic quantum number, occupation probability) SP energy, $(m_\lambda^\pi, P_\lambda)e_\lambda$, MeV	$(1/2^+, 1.0)$ -46.2	$(1/2^+, 0.81)$ -35.4	$(1/2^+, 1.0)$ -43.9	$(1/2^+, 0.84)$ -34.5	(+) -28	35.5±1.0
	$(3/2^-, 1.0)$ -23.2	$(3/2^-, 0.82)$ -16.5	$(3/2^-, 1.0)$ -21.8	$(3/2^-, 0.84)$ -15.9	(-) -15	15.0±0.5
	$(1/2^-, 1.0)$ -21.8	$(1/2^-, 0.83)$ -15.5	$(1/2^-, 1.0)$ -20.4	$(1/2^-, 0.85)$ -14.9	(-) -15	15.0±0.5
Shift parameter, C, MeV	48.64	46.84	40.76	38.76		

[a]The orbital energies in the ZR column refer to proton separation energies.
[b]Proton separation energies from ref. 42.

It has been known for some time that ordinary HF calculations do not provide good results for separation energies, and we observe that the ZR calculation differs by 20 percent from experiment for the most tightly bound state, although the p states are well represented. This is somewhat better than previous HF predictions of SP energies, however, since one usually finds that at least one of the predicted SP energies differs significantly (~50 percent) from experiment. Unrenormalized BHF is known to suffer from large rearrangement corrections and is also not expected to agree very well with experiment; this too is observed. The RBHF results are clearly the best of the three calculations.

An analogue of Koopmans' theorem for separation energies in RBHF has recently been established (ref. 34). Thus we may associate the energy of the first unoccupied

orbit in ^{12}C with the separation energy of the last neutron in ^{13}C. Experimentally, the number is found to be 4.95 MeV (ref. 41). The RBHF prediction is about 4.5 MeV, which is in good agreement with the measured result.

Table II contains the results for ^{16}O. Since these results are for spherical ^{16}O and this case has been discussed extensively in other studies, we do not feel it necessary to go into detail here. Efforts to obtain the deformed excited state in ^{16}O have thus far been unsuccessful. This state has been found, in standard HF studies, to be a 4p-4h state lying some 20 to 25 MeV above the ground state. The 4p-4h states which we have investigated so far have been very unstable.

TABLE II. - COMPARISON OF BHF AND RBHF PREDICTIONS OF INTRINSIC PROPERTIES OF ^{16}O WITH THE RESULTS OF ŽOFKA AND RIPKA

Properties	Oscillator length, b, fm					
	1.57		1.77		1.67	
	Calculation					
	BHF	RBHF	BHF	RBHF	ZR[a]	Experiment[b]
Binding energy per nucleon, -E/A, MeV	6.2	7.3	6.2	7.0	7.5	7.98
Energy gap, Δ, MeV	18.0	16.9	17.2	15.8	17.0	
Intrinsic rms radius, $\langle R^2 \rangle^{1/2}$, fm	2.16	2.27	2.25	2.33	2.72	2.67±0.03
Mass quadrupole moment, $\langle Q_2 \rangle$, fm^2	0.0	0.0	0.0	0.0	0.0	
Mass hexadecapole moment, $\langle Q_4 \rangle$, fm^4	0.0	0.0	0.0	0.0	0.0	
(Magnetic quantum number, occupation probability) SP energy, $(m_\lambda^\pi, P_\lambda)\ e_\lambda$, MeV	$(1/2^+, 1.0)$ -56.7	$(1/2^+, 0.79)$ -43.9	$(1/2^+, 1.0)$ -54.9	$(1/2^+, 0.83)$ -43.6	(+) -33	43±5
	$(3.2^-, 1.0)$ -30.0	$(3/2^-, 0.80)$ -21.3	$(3/2^-, 1.0)$ -28.3	$(3/2^-, 0.82)$ -20.9	(-) -17	21.8
	$(1/2^-, 1.0)$ -30.0	$(1/2^-, 0.80)$ -21.3	$(1/2^-, 1.0)$ -28.3	$(1/2^-, 0.82)$ -20.9	(-) -17	21.8
	$(1/2^-, 1.0)$ -23.5	$(1/2^-, 0.80)$ -17.0	$(1/2^-, 1.0)$ -22.6	$(1/2^-, 0.82)$ -16.8	(-) -17	15.7
Shift parameter, C, MeV	50.46	48.64	42.62	40.64		

[a]The orbital energies in the ZR column refer to proton separation energies.

[b]Proton separation energies from ref. 33.

Of the even Z-even N nuclei in the s-d shell, ^{20}Ne has proved most amenable to description by means of standard HF theory. The various properties of this nucleus that are primarily long range in character may be obtained with good accuracy from a prolate intrinsic state with a rather large hexadecapole moment. Those properties primarily short range in character (particularly the binding energy) are of course poorly given in standard HF theory.

Our BHF and RBHF results for ^{20}Ne are presented in table III. All the calculations

TABLE III. - COMPARISON OF BHF AND RBHF PREDICTIONS OF INTRINSIC PROPERTIES OF ^{20}Ne WITH THE RESULTS OF ŽOFKA AND RIPKA

Properties	Oscillator length, b, fm					
	1.57		1.77		1.83	
	Calculation					
	BHF	RBHF	BHF	RBHF	ZR[a]	Experiment[b]
Binding energy per nucleon, -E/A, MeV	5.8	7.1	6.1	6.9	7.1	8.2
Energy, Δ, MeV	7.6	7.4	7.9	7.5	6.9	
Intrinsic rms radius, $\langle R^2 \rangle^{1/2}$, fm	2.36	2.47	2.46	2.56	3.05	(2.55)[b]
Mass quadrupole moment, $\langle Q_2 \rangle$, fm^2	56.9	63.6	62.1	70.4	97.9	(69.8)[b]
Mass hexadecapole moment, $\langle Q_4 \rangle$, fm^4	130.7	150.7	167.7	202.9		(214)[b]
(Magnetic quantum number, occupation probability) SP energy, (m_λ, P_λ), e_λ, MeV	$(1/2^+, 1.0)$ -61.5	$(1/2^+, 0.79)$ -47.7	$(1/2^+, 1.0)$ -59.9	$(1/2^+, 0.83)$ -47.3	(+) -34	
	$(1/2^-, 1.0)$ -38.2	$(1/2^-, 0.79)$ -28.3	$(1/2^-, 1.0)$ -36.5	$(1/2^-, 0.81)$ -27.8	(-) -23	
	$(3/2^-, 1.0)$ -32.1	$(3/2^-, 0.80)$ -22.8	$(3/2^-, 1.0)$ -30.3	$(3/2^-, 0.82)$ -22.3	(-) -17	
	$(1/2^-, 1.0)$ -27.1	$(1/2^-, 0.79)$ -19.4	$(1/2^-, 1.0)$ -25.9	$(1/2^-, 0.81)$ -19.1	(-) -17	
	$(1/2^+, 1.0)$ -18.6	$(1/2^+, 0.81)$ -12.0	$(1/2^+, 1.0)$ -18.4	$(1/2^+, 0.82)$ -12.6	(+) -10	
Shift parameter, C, MeV	50.46	48.63	42.62	40.64		

[a] Proton separation energies as referred to in table I.

[b] These values are obtained from a standard HF calculation with b = 1.77 fm.

yield prolate intrinsic states with the same level ordering, and as for the preceding cases, the RBHF predictions are somewhat more satisfactory than the BHF. The rms radius of ^{20}Ne has not been measured, but it can be inferred from measurements on neighboring nuclei to be about 2.8 femtometers. Again we find the BHF and RBHF radius values too small, and the ZR value somewhat too large, and all the calculations underbind the nucleus.

CONCLUDING REMARKS

In spite of the deficiencies found in BHF and RBHF calculations for spherical nuclei, it is gratifying to note that the successful features of the standard HF theory seem also to be present in the Brueckner version. As evidence of this, we may observe the similarity of values for the "long-range observables" $\langle R^2 \rangle^{1/2}$, Q_2, and Q_4, which are listed in table III for both HF and RBHF calculations with b = 1.77 femtometers. If this similarity persists for other values of the oscillator parameter, one would be justified in claiming that renormalized Brueckner-Hartree-Fock calculations give a fundamental description of ^{20}Ne with a consistent degree of accuracy for all its observables. Hopefully this would give even greater emphasis to investigation of the chief drawback of BHF and RBHF theory, namely, the persistent prediction of nuclei which are too small and too loosely bound.

When the Pauli corrections are made as suggested in the THEORY section, indications are that one may obtain some improvement in binding but that the radius will be less affected. Evaluation of the rms radius using a correlated wavefunction would likely yield a more reliable prediction.

Lewis Research Center,
National Aeronautics and Space Administration,
Cleveland, Ohio, April 7, 1972,
112-02.

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